STRESSES AND DEFORMATIONS DUE TO TANGENTIAL AND NORMAL LOADS ON AN ELASTIC SOLID WITH APPLICATIONS TO CONTACT STRESSES

BY

CHANG KENG LIU

B.S., Chiao-Tung University, 1943
M.S., University of Illinois, 1946

AN ABSTRACT OF A THESIS

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Introduction

The problem of determining the stresses and deformations within an elastic solid which is subjected to a given force across its surface is of interest in the field of practical design. There are many instances in machine design where the stresses due to the contact between members are regarded as a criterion of design. Ball and roller bearings, valve tappets and gear teeth are among the common examples.

Many solutions have been found for the contact stresses due to the action of a normal force distributed over a portion of the surface of the solid, but all have neglected the possibility of a frictional or tangential force acting simultaneously between the contact surfaces. The present investigation is to obtain a solution for the stresses in the elastic solid due to a combination of normal and tangential forces distributed over the contact surface. Two approaches to the problem are made, first the problem is treated as a three-dimensional one, and second it is treated as a two-dimensional one.

Discussion

The distribution of the tangential force over the contact area is assumed to be the Hertzian distribution, that is, it is of the same nature as the distribution of the normal force over the same area, except that it bears a linear factor of proportionality which has a physical significance of coefficient of friction. This assumption is based on the fact that the frictional force can not exceed the product of the normal load at the same point and the coefficient of friction of the sliding surfaces concerned. The distribution of the tangential force

used in this investigation therefore corresponds to the one in which slip has progressed over the entire contact area.

Contact under load between gear teeth and such contact between roller bearings illustrate a plane stress and a plane strain problem, respectively. These two problems are treated in detail and the principal stresses corresponding to the respective cases are plotted. From the three principal stresses found, the three maximum shearing stresses are obtained by taking half of the difference of any two principal stresses at a time. One of these maximum shearing stresses is the so-called subsurface shear.

Previous investigators have found that the shearing stress on the axis of symmetry produced by normal load distributed on the surface of the elastic body is maximum not on the surface of the body but at a distance below the surface. For two decades, this discovery had led engineers to believe that the failures which occurred in mating machine parts, such as cracks and pits, were initiated in the interior of the body and spread toward the surface after a number of repetitions of the loading. However, in most instances, cracks and pits have been observed on the surface of the material, but, when the cross section of the body is examined under a microscope, no sign of damage is located below the surface. This investigation, in which both tangential and normal forces over the contact area are taken into consideration, has revealed that the maximum shearing stress is very likely to occur at the surface instead of at a subsurface point as found by Thomas and Hoersch for a distributed normal force only. Furthermore, it has been shown that there are large tension and compression stresses at points in the surface which may cause the surface crarks.

Conclusion

In this thesis the stresses and deformations in a body due to an elliptical distribution of normal and tangential forces acting over a small area on the surface of the body have been

found. Such a loading finds practical application to sliding contact between machine parts, such as gear teeth, roller bearings, ball bearings, tappets, etc.

A. Two-dimensional problem.

1. A complete solution for stresses has been presented for the plane state of stress and plane state of strain.

2. The principal stresses and the principal shearing stress have been found for a plane state of stress and for a plane state of strain on the assumption that the coefficient of friction is equal to one-third. It also has been found that the values of shearing stress at the surface are considerably larger than the maxium subsurface shear.

3. The existence of stresses described above at the surface makes possible a plausible explanation of the formation of pitting cracks at the surface of members subjected to sliding contact stresses.

B. Three-dimensional problem.

1. A solution for the stresses and deformations in a body due to an elliptical distribution of tangential force acting on a portion of the surface of the body has been found. This solution is in the form of integrals.

The author feels that for further study of this problem of contact under tangential and normal loads the following phases of the problem are of great importance:

A. An experimental study of the problem of contact stresses to determine the effect of the stresses caused by tangential contact loads (due to friction) on pitting strength of rollers, gear teeth, etc.

B. Determination of the coefficient of friction for the sliding surfaces concerned.

C. A careful study of the actual nature of sliding contact of hard and soft metallic surfaces.

D. A feasible way of evaluating the integrals in the solution of the three-dimensional problem.

VITA

Chang Keng Liu was born on March 28, 1922, in Soochow, Kiangsu, China, where he lived until 1938 when the Marco Polo Bridge Incident occurred. At that time, having finished the first year of his high school education at the Soochow Provincial Middle school, he and his family moved to Shanghai. He then enrolled in the Shanghai Provincial Middle School for the second year of his high school curriculum. As the war raged near to the site of the school, he transferred to the Yangchow Provincial Middle School which was then located in the downtown section of Shanghai. He graduated from the Yangchow Provincial Middle School in 1939, and enrolled in the National Chiao-Tung University in Shanghai. Upon finishing his junior year he joined the movement of the University inland toward free China. He rejoined the University in Kiuloonpou, Chungking, in the winter of 1942, and graduated in 1943 among the first class of that University in Chungking. In December, 1943, he took part in the examination offered by the Ministry of Education selecting

students for advanced studies abroad, and was qualified among those who intended to pursue their studies in the United States of America.

He began his graduate work at the University of Illinois in 1945 under the supervision of the late Professor Everett G. Young, who had been adviser to the Ministry of Communications of the Chinese government from 1918 to 1925. After he received his Master of Science degree in Railway Mechanical Engineering in 1946, he applied his training at the American Locomotive Company, Schenectady, New York. In 1947, he returned to the University of Illinois for further graduate study in Theoretical and Applied Mechanics. He has held a Research Assistantship in that Department from September, 1947, till now.

He was elected a member of Pi Mu Epsilon Mathematics Honorary Society, and an associate member of Sigma Xi in April, 1946. He has been a junior member of the American Society of Mechanical Engineers since 1947.

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